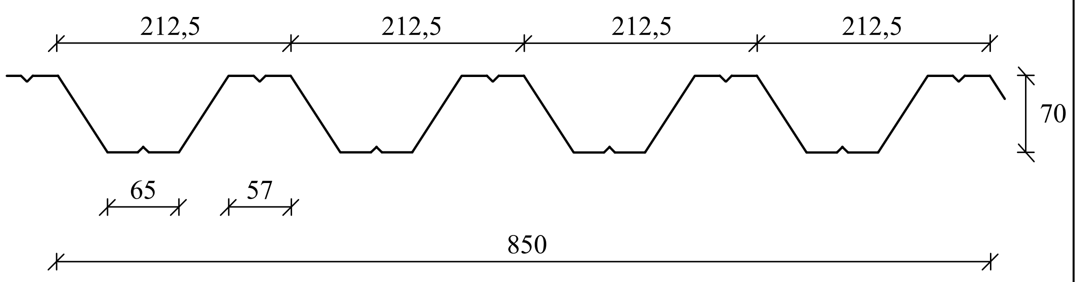
DESIGN EXAMPLE 11 – DESIGN OF A TWO-SPAN TRAPEZOIDAL ROOF SHEETING

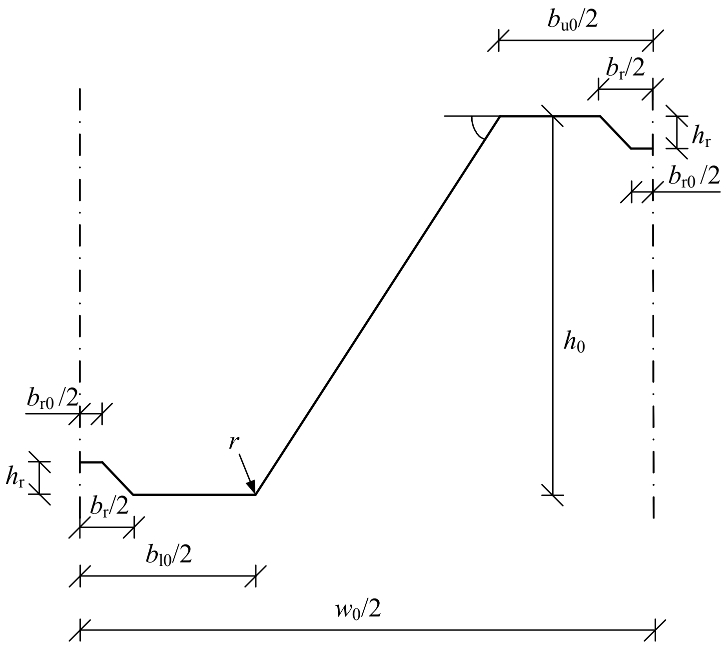
This example deals with a two-span trapezoidal roof sheeting with a thickness of 0,6 mm from stainless steel grade 1.4401 CP500, i.e. cold worked to a yield strength of 500 N/mm^2. Comparisons will be made to a similar sheeting of grade 1.4401 in the annealed condition, i.e. f_y = 240 N/mm^2 (see also Design Example 3).

If the nominal yield strength in all directions of the sheet is not guaranteed by the producer it should be reduced to 80% of its value. In this example it is assumed that the strength was not guaranteed in order to demonstrate this. Section 3.2.4

The dimensions of the roof sheeting are shown below.



A detailed sketch of the roof sheeting is given in the figure below. The lower flange will be in compression over the mid support and therefore this case will be checked in this example.



Mid line dimensions:

h_0 = 70 mm
w_0 = 212,5 mm
b_{u0} = 57 mm
b_r = 20 mm
h_r = 6 mm
b_{r0} = 8 mm
b_{l0} = 65 mm

radius, r = 3 mm

angle, φ = 57,1°

Data

Span length	$L = 3{,}5$ m	
Load	$q = 1{,}4$ kN/m^2	
Self weight	$g = 0{,}07$ kN/m^2	
Sheeting thickness	$t = 0{,}6$ mm	
Width of support	$s_s = 100$ mm	
Yield strength	$f_y = 0{,}8 \times 500 = 400$ N/mm^2	Table 3.5
Modulus of elasticity	$E = 200\,000$ N/mm^2	
Partial factor	$\gamma_{M0} = 1{,}1$	Table 2.1
Partial factor	$\gamma_{M1} = 1{,}1$	Table 2.1
Load factor	$\gamma_G = 1{,}35$ (permanent loads)	Section 2.3.2
Load factor	$\gamma_Q = 1{,}5$ (variable loads)	Section 2.3.2

Effective section properties

Maximum width-to-thickness ratios — Table 4.1

$$\max(b_{l0}/t,\quad b_{u0}/t) = b_{l0}/t = 108 < 400$$

$$h_0/t = 117 < 400$$

Location of the centroidal axis when the web is fully effective

Effective width of the compression flange — Section 4.4.1

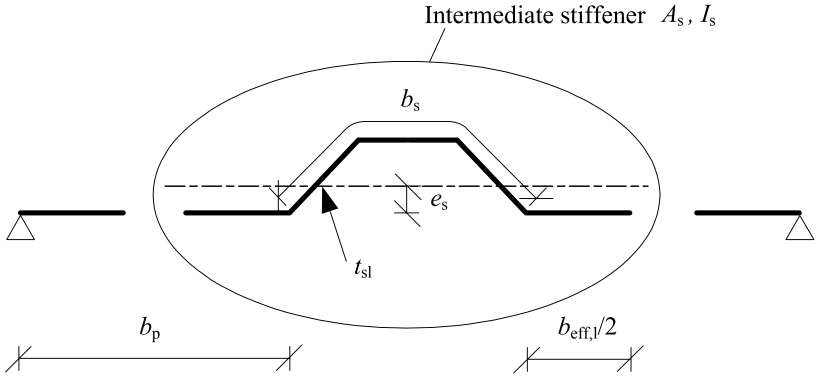
$$b_p = \frac{b_{l0} - b_r}{2} = 22,5 \text{ mm} \qquad \varepsilon = \sqrt{\frac{235}{f_y}\frac{E}{210000}} = 0,75$$ Table 4.2

$$k_\sigma = 4 \qquad \bar{\lambda}_p = \frac{b_p/t}{28,4\varepsilon\sqrt{k_\sigma}} = 0,883$$ Table 4.3, Eq. 4.2

$$\rho = \frac{0,772}{\bar{\lambda}_p} - \frac{0,125}{\bar{\lambda}_p^2} = 0,714 \rightarrow \quad b_{\text{eff,l}} = \rho b_p = 16,1 \text{ mm}$$ Eq. 4.1a, Table 4.3

Reduced thickness of the flange stiffener: — Section 4.5.3

The lower compressed flange is shown in detail below.



Effective thickness of the inclined part of the stiffener

$$t_{\mathrm{rl}} = \frac{\left(\sqrt{\left(\frac{b_{\mathrm{r}} - b_{\mathrm{r0}}}{2}\right)^2 + h_{\mathrm{r}}^2}\right)t}{h_{\mathrm{r}}} = 0,85 \text{ mm}$$

$$A_{\mathrm{s}} = (b_{\mathrm{eff,l}} + b_{\mathrm{r0}})t + 2h_{\mathrm{r}}t_{\mathrm{rl}} = 24,62 \text{ mm}^2$$ Figure 4.3

$$e_{\mathrm{s}} = \frac{b_{\mathrm{r0}}h_{\mathrm{r}}t + 2h_{\mathrm{r}}\frac{h_{\mathrm{r}}}{2}t_{\mathrm{rl}}}{A_{\mathrm{s}}} = 2,41 \text{ mm}$$

The second moment of area for the stiffener is calculated with two strips of width $15t$ adjacent to the stiffener (smaller terms neglected) — Figure 4.3

$$I_{\mathrm{s}} = 2\times 15t^2e_{\mathrm{s}}^2 + b_{\mathrm{r0}}t(h_{\mathrm{r}} - e_{\mathrm{s}})^2 + 2h_{\mathrm{r}}t_{\mathrm{rl}}\left(\frac{h_{\mathrm{r}}}{2} - e_{\mathrm{s}}\right)^2 + 2\frac{t_{\mathrm{rl}}h_{\mathrm{r}}^3}{12} = 159,1 \text{ mm}^4$$

$$b_{\mathrm{s}} = 2\sqrt{h_{\mathrm{r}}^2 + \left(\frac{b_{\mathrm{r}} - b_{\mathrm{r0}}}{2}\right)^2} + b_{\mathrm{r0}} = 24,97 \text{ mm}$$

$$l_{\mathrm{b}} = 3,07\sqrt[4]{\frac{I_{\mathrm{s}}b_{\mathrm{p}}^2(2b_{\mathrm{p}} + 3b_{\mathrm{s}})}{t^3}} = 251,0 \text{ mm}$$ Eq. 4.9

$$s_{\mathrm{w}} = \sqrt{\left(\frac{w_0 - b_{\mathrm{u0}} - b_{\mathrm{l0}}}{2}\right)^2 + h_0^2} = 83,4 \text{ mm}$$

$$b_{\mathrm{d}} = 2b_{\mathrm{p}} + b_{\mathrm{s}} = 70,0 \text{ mm}$$ Eq. 4.11

$$k_{\mathrm{wo}} = \sqrt{\frac{s_{\mathrm{w}} + 2b_{\mathrm{d}}}{s_{\mathrm{w}} + 0,5b_{\mathrm{d}}}} = 1,37$$ Eq. 4.10

$$l_{\mathrm{b}}/s_{\mathrm{w}} = 3,01 > 2 \rightarrow \quad k_{\mathrm{w}} = k_{\mathrm{wo}} = 1,37$$ Eq. 4.7

$$\sigma_{\mathrm{cr,s}} = \frac{4,2k_{\mathrm{w}}E}{A_{\mathrm{s}}}\sqrt{\frac{I_{\mathrm{s}}t^3}{4b_{\mathrm{p}}^2(2b_{\mathrm{p}} + 3b_{\mathrm{s}})}} = 557,5 \text{ N/mm}^2$$ Eq. 4.3

$$\bar{\lambda}_{\mathrm{d}} = \sqrt{\frac{f_{\mathrm{y}}}{\sigma_{\mathrm{cr,s}}}} = 0,85 \quad \rightarrow \quad \chi_{\mathrm{d}} = 1,47 - 0,723\bar{\lambda}_{\mathrm{d}} = 0,86$$ Eq. 4.16

$$t_{\mathrm{red}} = \chi_{\mathrm{d}}t = 0,51 \text{ mm}$$

Optionally iterate to refine the value of the reduction factor for buckling of the stiffener. — prEN 1993-1-3, clause 5.5.3.3 (3)

Distance to the neutral axis from the compressed flange (fully effective web)

$$A_{\text{tot}} = \sum A_i = 84,0\ \text{mm}^2$$

$$e_{\text{c}} = \frac{\sum A_i e_i}{A_{\text{tot}}} = 36,3\ \text{mm}$$

Effective cross-section of the web — Section 4.4.1

$$\psi = -\frac{h_0 - e_{\text{c}}}{e_{\text{c}}} = -0,929 \qquad k_\sigma = 7,81 - 6,29\psi + 9,78\psi^2 = 22,1$$ — Table 4.3

$$b_{\text{p,w}} = s_{\text{w}} = 83,4\ \text{mm} \qquad \bar{\lambda}_{\text{p}} = \frac{b_{\text{p,w}}/t}{28,4\varepsilon\sqrt{k_\sigma}} = 1,391$$ — Eq. 4.2

$$\rho = \frac{0,772}{\bar{\lambda}_{\text{p}}} - \frac{0,125}{\bar{\lambda}_{\text{p}}^2} = 0,490 \ \rightarrow \ b_{\text{eff,w}} = \rho\frac{b_{\text{p,w}}}{1-\psi} = 21,2\ \text{mm}$$ — Eq. 4.1a, Table 4.3

$$s_{\text{eff,1}} = 0,4 b_{\text{eff,w}} = 8,47\ \text{mm} \qquad s_{\text{eff,2}} = 0,6 b_{\text{eff,w}} = 12,7\ \text{mm}$$ — Table 4.3

Effective cross section properties per half corrugation

$$A_{\text{eff,tot}} = \sum A_{\text{eff},i} = 70,8\ \text{mm}^2$$

$$e_{\text{eff,c}} = \frac{\sum A_{\text{eff},i} e_{\text{eff},i}}{A_{\text{eff,tot}}} = 40,0\ \text{mm}$$

$$I_{\text{tot}} = \sum I_{\text{eff,i}} + \sum A_{\text{eff,i}}\left(e_{\text{c}} - e_{\text{eff,i}}\right)^2 = 51710\ \text{mm}^4$$

Bending resistance per unit width (1m) — Section 4.7.4

$$I = I_{\text{tot}}\frac{1000\ \text{mm}}{0,5 w_0} = 486685\ \text{mm}^4$$

$$W_{\text{eff,l}} = \frac{I}{e_{\text{c}}} = 12165\ \text{mm}^3 \qquad W_{\text{eff,u}} = \frac{I}{h_0 - e_{\text{c}}} = 16227\ \text{mm}^3$$

$W_{\text{eff,l}} < W_{\text{eff,u}} \rightarrow W_{\text{eff,min}} = W_{\text{eff,l}}$

$$M_{\text{c,Rd}} = W_{\text{eff,min}} f_{\text{y}} / \gamma_{\text{M0}} = 4,42\ \text{kNm}$$ — Eq. 4.29

Resistance to local transverse forces at intermediate support

Resistance to local transverse forces per unit width (1 m)

α = 0,15 (for sheeting profiles) and $l_a = s_s$ — prEN 1993-1-3, Eq. 6.20c, 6.19b and 6.18

$$R_{w,Rd} = \alpha t^2 \sqrt{f_y E}\left(1-0,1\sqrt{r/t}\right)\left[0,5+\sqrt{0,02 l_a/t}\right]\left(2,4+\left(\varphi/90\right)^2\right)\frac{1000\text{ mm}}{0,5w_0}\Big/\gamma_{M1}$$

$R_{w,Rd}$ = 20,9 kN

Interaction between bending moment and transverse force

The maximum bending moment will appear at the intermediate support where it will interact with the support reaction and therefore the following checks must be performed.

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1 \qquad \frac{F_{Ed}}{R_{w,Rd}} \leq 1 \qquad \frac{M_{Ed}}{M_{c,Rd}} + \frac{F_{Ed}}{R_{w,Rd}} \leq 1,25$$

prEN 1993-1-3, Eqs. 6.28a-c

Design load per unit width (1 m) — Section 2.3.2

$$q_d = \gamma_G g + \gamma_Q q = 2,20\text{ kN/m}$$

Eq. 2.3

The design load, q_d, gives the following bending moment and support reaction at the intermediate support.

$$M_{Ed} = \frac{qL^2}{8} = 3,37\text{ kNm} \qquad F_{Ed} = \frac{5}{4}qL = 9,63\text{ kN}$$

$$\frac{M_{Ed}}{M_{c,Rd}} = 0,76 \qquad \frac{F_{Ed}}{R_{w,Rd}} = 0,46 \qquad \frac{M_{Ed}}{M_{c,Rd}} + \frac{F_{Ed}}{R_{w,Rd}} = 1,22$$

OK

Deflection at serviceability limit state

For verification in the serviceability limit state the effective width of compression elements should be based on the compressive stress in the element under serviceability limit state loading. The maximum compression stress is calculated as follows. A conservative approximation is made based on $W_{eff,min}$ from ultimate limit state.

prEN 1993-1-3, clause 5.5.1(4)

$$M_{Ed,ser} = \frac{(q+g)L^2}{8} = 2,25\text{ kNm}$$

Section 2.3.4

$$\sigma_{com,Ed,ser} = \frac{M_{Ed,ser}}{W_{eff,min}} = 186\text{ N/mm}^2$$

Now, the effective section properties are determined as before but with f_y replaced by $\sigma_{com,Ed,ser}$. The calculations will not be shown here but the interesting results are:

I = 573 150 mm^4
W_u = 15 866 mm^3
W_l = 16 919 mm^3

Determination of the deflection:

Appendix C

Secant modulus corresponding to the stresses in the tension and compression flange respectively.

$$\sigma_{1,\text{Ed,ser}} = \frac{M_{\text{Ed,ser}}}{W_{\text{u}}} = 142 \text{ N/mm}^2$$

$$\sigma_{2,\text{Ed,ser}} = \frac{M_{\text{Ed,ser}}}{W_{\text{l}}} = 133 \text{ N/mm}^2$$

$$E_{\text{s},1} = \frac{E}{1 + 0,002\dfrac{E}{f_{\text{y}}}\left(\dfrac{\sigma_{1,\text{Ed,ser}}}{f_{\text{y}}}\right)^{n-1}} = 199\ 604 \text{ N/mm}^2 \qquad n = 7,0$$

Appendix C Table C.1

$$E_{\text{s},2} = \frac{E}{1 + 0,002\dfrac{E}{f_{\text{y}}}\left(\dfrac{\sigma_{2,\text{Ed,ser}}}{f_{\text{y}}}\right)^{n-1}} = 199\ 730 \text{ N/mm}^2$$

$$E_{\text{s}} = \frac{E_{\text{s},1} + E_{\text{s},2}}{2} = 199\ 667 \text{ N/mm}^2$$

As a simplification, the variation of E_{s} along the length of the member may be neglected and the minimum value of E_{s} of that member may conservatively be used throughout its length, i.e.

Appendix C

$E_{\text{s}} = E_{\text{s},1} = 199\ 603 \text{ N/mm}^2$

The permitted deflection is L/300 = 11,7 mm

$$x = \frac{1+\sqrt{33}}{16} L = 1,47 \text{ m} \qquad \text{(location of maximum deflection)}$$

$$\delta = \frac{(g+q)L^4}{48 E_{\text{s},1} I}\left(\frac{x}{L} - 3\frac{x^3}{L^3} + 2\frac{x^4}{L^4}\right) = 10,4 \text{ mm} \qquad \textbf{OK}$$

Comparison with sheeting in grade 1.4401 in the annealed condition

The bending resistance per unit width of identical sheeting in grade 1.4401 in the annealed condition (f_{y} = 240 N/mm²) is:

$M_{\text{c,Rd}}$ = 3,22 kNm

and the resistance to local transverse forces is:

$R_{\text{w,Rd}}$ = 16,2 kN

With sheeting made from grade 1.4401 in the annealed condition, the span must be reduced to 2,9 m compared to 3,5 m for material in the cold worked strength condition. Hence, sheeting made from cold worked material enables the span to be increased, meaning that the number of secondary beams or purlins could be reduced, leading to cost reductions.