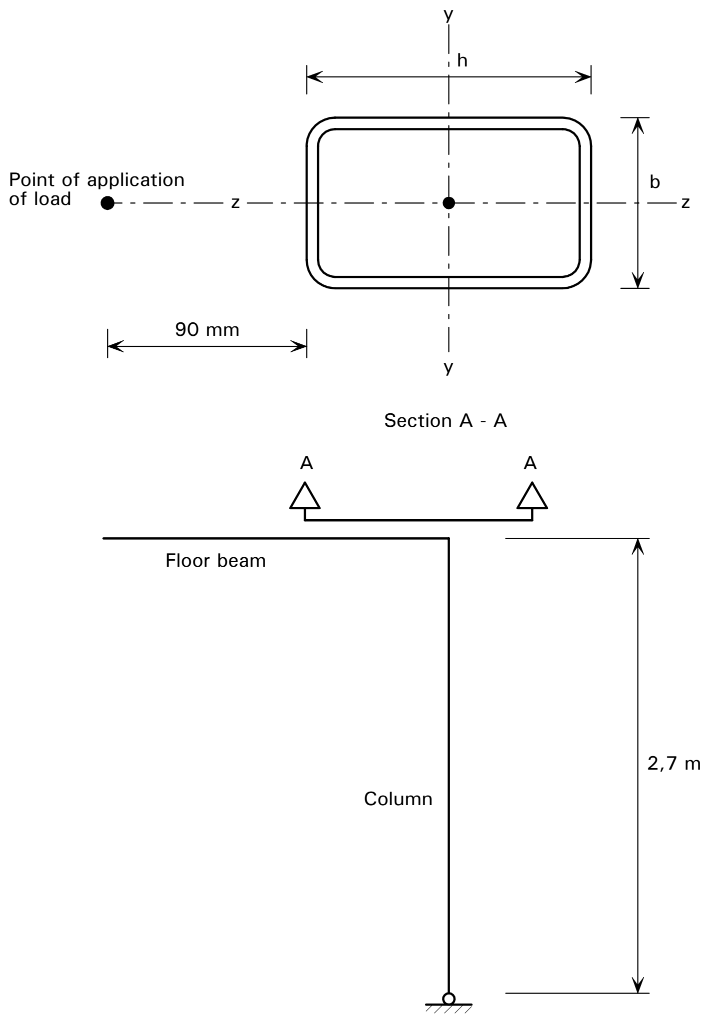
DESIGN EXAMPLE 10 - AXIALLY LOADED COLUMN IN FIRE

Design an unprotected rectangular hollow section subject to axial load and bending for 30 minutes fire resistance.

The column length is 2,7 m and is subject to axial load from the end reaction of a floor beam at an eccentricity of 90 mm from the narrow face of the column.



Actions

This eccentricity is taken to be 90 mm + $h/2$, where h is the depth of the section. Thus the beam introduces a bending moment about the column's major axis.

The unfactored actions are:	Permanent action:	6 kN
	Variable action:	7 kN

The column will initially be checked at the ultimate limit state (LC1) and subsequently at the fire limit state (LC2) for a fire duration of 30 minutes. The loadcases are as follows:

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CALCULATION SHEET

LC1 (ultimate limit state) $\sum_j \gamma_{G,j} G_{k,j} + \gamma_{Q,1} Q_{k,1}$ — Eqn. 2.3

$\gamma_{G,j}$ = 1,35 (unfavourable effects) — Section 2.3.2
$\gamma_{Q,1}$ = 1,5

LC2 (fire limit state) $\sum_j \gamma_{GA,j} G_{k,j} + \psi_{1,1} Q_{k,1}$

γ_{GA} = 1,0

Values for $\psi_{1,1}$ are given in EN 1990 and NA for EN 1990, but for this example conservatively assume $\psi_{1,1}$ = 1,0

Design at the Ultimate Limit State (LC1)

Loading on the corner column due to shear force at end of beam (LC1):

Axial force N_{Ed} = 1,35 × 6 +1,5 × 7 = 18,6 kN

Try 100 × 50 × 6 Rectangular Hollow Section

Major axis bending moment (due to eccentricity of shear force from centroid of column),
$M_{y,Ed}$ = 18,6 × (0,09+0,10/2) = 2,60 kNm

Material properties

Use material grade 1.4401

0,2% proof stress = 220 N/mm^2 and f_u = 530 N/mm^2 — Table 3.1

Take f_y as the 0,2% proof stress = 220 N/mm^2 — Section 3.2.4

E = 200 000 N/mm^2 and G = 76 900 N/mm^2 — Section 3.2.4

Cross-section properties – 100 x 50 x 6 mm RHS

$W_{el,y}$ = 32,58 × 10^3 mm^3 i_y = 32,9 mm
$W_{pl,y}$ = 43,75 × 10^3 mm^3 i_z = 19,1 mm
A_g = 1500 mm^2 t = 6 mm

Classification of the cross-section

ε = 1,01 — Table 4.2

Assume conservatively that $c = h - 2t$ = 100 – 12 = 88 mm for web

Webs subject to compression: $\frac{c}{t} = \frac{88}{6}$ = 14,7

For Class 1, $\frac{c}{t} \leq 25{,}7\varepsilon$ = 25,96 ∴ Web is Class 1 — Table 4.2

By inspection, if the web is Class 1 subject to compression, then the flange will also be Class 1. ∴ Cross-section is Class 1

Partial safety factors

The following partial safety factors are used throughout the design example for LC1: — Table 2.1

γ_{M0} = 1,1

γ_{M1} = 1,1

Buckling resistance to axial compression — Section 5.3.3

Resistance to flexural buckling about the *z-z* axis:

$N_{b,z,Rd} = \dfrac{\chi_z A_g f_y}{\gamma_{M1}}$ For Class 1, 2 and 3 cross-sections — Eq. 5.2a

χ = reduction factor to account for buckling = $\dfrac{1}{\varphi + \left[\varphi^2 - \bar{\lambda}^2\right]^{0,5}} \leq 1$ — Eq. 5.3

$\varphi = 0,5\left(1 + \alpha(\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2\right)$ — Eq. 5.4

$\bar{\lambda}_z = \dfrac{L_{cr}}{i_z}\dfrac{1}{\pi}\sqrt{\dfrac{f_y}{E}}$ — Eq. 5.5a

L_{cr} = buckling length of column, taken conservatively as 1,0 × column length = 2,7 m

$\bar{\lambda}_z = \dfrac{2700}{19,1}\dfrac{1}{\pi}\sqrt{\dfrac{220}{200000}} = 1,492$

For hollow sections subject to flexural buckling, α = 0,49 and $\bar{\lambda}_0$ = 0,40 — Table 5.1

$\varphi = 0,5\left(1 + 0,49(1,492 - 0,4) + 1,492^2\right) = 1,881$

$\chi_z = \dfrac{1}{1,881 + \left[1,881^2 - 1,492^2\right]^{0,5}} \leq 1$

χ_z = 0,3305

$N_{b,z,Rd} = \dfrac{0,3305 \times 1500 \times 220}{1,1} = 99,15$ kN

(Resistance to torsional buckling will not be critical for a rectangular hollow section with a *h*/*b* ratio of 2.) — Section 5.3.1

N_{Ed} = 18,6 kN Buckling resistance of cross-section is OK

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CALCULATION SHEET

Axial compression and bending resistance

Cross sectional resistance interaction check — Section 4.7.6; prEN 1993-1-3, Clause 6.1.9

$$\frac{N_{\mathrm{Ed}}}{N_{\mathrm{c,Rd}}} + \frac{M_{\mathrm{y,Ed}} + N_{\mathrm{Ed}}\, e_{\mathrm{Ny}}}{M_{\mathrm{c,y,Rd}}} + \frac{M_{\mathrm{z,Ed}} + N_{\mathrm{Ed}}\, e_{\mathrm{Nz}}}{M_{\mathrm{c,z,Rd}}} \leq 1$$

Eq. 4.25

$$\mathrm{N}_{\mathrm{c,Rd}} = \frac{A_{\mathrm{g}} f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} = \frac{1500 \times 220}{1,1} = 300 \text{ kN}$$

$e_{\mathrm{Ny}} = e_{\mathrm{NZ}} = 0$

$M_{\mathrm{z,Ed}} = 0$

Eq. 4.27

$$M_{\mathrm{c,y,Rd}} = \frac{W_{pl,y} f_{\mathrm{y}}}{\gamma_{M0}} = \frac{43,75 \times 10^3 \times 220}{1,1} = 8,75 \text{ kNm}$$

$$\frac{N_{\mathrm{Ed}}}{N_{\mathrm{c,Rd}}} + \frac{M_{\mathrm{y,Ed}}}{M_{\mathrm{c,y,Rd}}} = \frac{18,6}{300} + \frac{2,60}{8,75} = 0,062 + 0,297 = 0,359 < 1,00$$

∴ Resistance of cross-section is OK

Buckling resistance interaction check — Section 5.5.2

Eq. 5.40

$$\frac{N_{\mathrm{Ed}}}{(N_{\mathrm{b,Rd}})_{\min}} + k_{\mathrm{y}}\left(\frac{M_{\mathrm{y,Ed}} + N_{\mathrm{Ed}}\, e_{\mathrm{Ny}}}{\beta_{\mathrm{W,y}} W_{\mathrm{pl,y}} f_{\mathrm{y}} / \gamma_{\mathrm{M1}}}\right) \leq 1$$

$\beta_{\mathrm{W,y}} = 1,0$ for Class 1 cross-sections

$$k_{\mathrm{y}} = 1,0 + 2\left(\bar{\lambda}_{\mathrm{y}} - 0.5\right)\frac{N_{\mathrm{Ed}}}{N_{\mathrm{b,Rd,y}}} \quad \text{but} \quad 1.2 \leq k_{\mathrm{y}} \leq 1,2 + 2\left(\frac{N_{\mathrm{Ed}}}{N_{\mathrm{b,Rd,y}}}\right)$$

Determine $N_{\mathrm{b,Rd,y}}$ using the same method used to calculate $N_{\mathrm{b,Rd,z}}$ given on sheet 3.

For hollow sections subject to flexural buckling, $\alpha = 0,49$ and $\bar{\lambda}_0 = 0,40$ — Table 5.1

$$\bar{\lambda}_y = \frac{L_{\mathrm{cr}}}{i_y}\frac{1}{\pi}\sqrt{\frac{f_y}{E}} = \frac{2700}{32,9}\frac{1}{\pi}\sqrt{\frac{220}{200000}} = 0,866$$

$$\varphi = 0,5\left(1 + 0,49(0,866 - 0,4) + 0,866^2\right) = 0,989$$

$$\chi_y = \frac{1}{0,989 + \left[0,989^2 - 0,866^2\right]^{0,5}} \leq 1$$

$\chi_y = 0,682 < 1,0$

$$N_{\mathrm{b,Rd,y}} = \frac{0,682 \times 1500 \times 220}{1,1} = 204,6 \text{ kN}$$

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$$k_{\mathrm{y}} = 1{,}0 + 2\left(\overline{\lambda}_{\mathrm{y}} - 0.5\right)\frac{N_{\mathrm{Ed}}}{N_{\mathrm{b,Rd,y}}}$$

$$k_{\mathrm{y}} = 1{,}0 + 2(0{,}866 - 0.5)\frac{18{,}6}{204{,}6} = 1{,}07 < 1{,}2$$

Therefore, $k_{\mathrm{y}} = 1{,}2$

$$\frac{N_{\mathrm{Ed}}}{(N_{\mathrm{b,Rd}})_{\mathrm{min}}} + k_{\mathrm{y}}\left(\frac{M_{\mathrm{y,Ed}} + N_{\mathrm{Ed}}\, e_{\mathrm{Ny}}}{\beta_{\mathrm{W,y}} W_{\mathrm{pl,y}}\, f_{\mathrm{y}} / \gamma_{\mathrm{M1}}}\right)$$

$$\frac{18{,}6}{99{,}15} + 1{,}2\left(\frac{2{,}60\times10^{6} + 0}{1{,}0\times 43{,}75\times10^{3}\times 220/1{,}1}\right) = 0{,}188 + 0{,}297 = 0{,}485 < 1{,}0$$

Thus member is OK for combined axial loading and uniaxial moment under LC1.

Design at the Fire Limit State (LC2)

For LC2, the column is designed for the following axial loads and moments.

Axial compressive force, $N_{\mathrm{fi,Ed}} = 1{,}0 \times 6 + 1{,}0 \times 7 = 13{,}0$ kN

Maximum bending moment $M_{\mathrm{y,fi,Ed}} = 13{,}0 \times (0{,}09+0{,}05) = 1{,}82$ kNm

Determine temperature in steel after 30 minutes fire duration — Section 7.4.7

Assume that the section is unprotected and that there is a uniform temperature distribution within the steel section. Increase in temperature during time interval Δt is found from:

$$\Delta\theta_{\mathrm{a,t}} = \frac{A_{\mathrm{m}}/V}{c_{\mathrm{a}}\rho_{\mathrm{a}}}\dot{h}_{\mathrm{net,d}}\Delta t \qquad \text{Eq. 7.34}$$

$$\dot{h}_{\mathrm{net,d}} = \dot{h}_{\mathrm{net,c}} + \dot{h}_{\mathrm{net,r}} \qquad \text{Eq. 7.35}$$

$$\dot{h}_{\mathrm{net,c}} = \alpha_{\mathrm{c}}\left(\theta_{\mathrm{g}} - \theta_{\mathrm{a}}\right) \qquad \text{Eq. 7.36}$$

Where:

θ_{g} = gas temperature of the environment of the member in fire exposure, given by the nominal temperature time curve:

θ_{g} = $20 + 345\log_{10}(8t + 1)$ — Eq. 7.38

θ_{a} = surface temperature of the member

$$\dot{h}_{\mathrm{net,r}} = \varphi\varepsilon_{\mathrm{res}} 5{,}67\times10^{-8}\left[\left(\theta_{\mathrm{g}} + 273\right)^{4} - \left(\theta_{\mathrm{a}} + 273\right)^{4}\right] \qquad \text{Eq. 7.37}$$

Initial input values for determination of final steel temperature are as follows:

A_{m}/V = 200 m^{-1}

α_{c} = 25 W/m^{2}K

Initial steel temperature, θ_a = 20°C

Resultant emissivity, ε_{res} = 0,2

Unit mass of stainless steel, ρ_a = 7850 kg/m^3

Configuration factor, φ = 1,0

The specific heat is temperature-dependent and is given by the following expression:

$c_a = 450 + 0{,}28\theta_a - 2{,}91 \times 10^{-4}\theta_a^2 + 1{,}34 \times 10^{-7}\theta_a^3$ J/kgK — Eq. 7.4

Δt = 2 seconds

The above formulae and initial input information were coded in an Excel spreadsheet and the following steel temperature, after a fire duration of 30 minutes, was obtained.

θ_a = 811°C

Reduction of mechanical properties at elevated temperature

The following reduction factors are required for calculation of resistance at elevated temperatures.

Young's modulus retention factor $k_{E,\theta} = E_\theta/E$

0,2% proof strength retention factor $k_{0,2proof,\theta} = f_{0,2proof,\theta}/f_y$

Ultimate tensile strength retention factor $k_{u,\theta} = f_{u,\theta}/f_u$

The value of the 2% yield strength at elevated temperature is also required for resistance calculations. This is given by the following expression:

$$f_{2,\theta} = f_{0,2proof,\theta} + g_{2,\theta}\left(f_{u,\theta} - f_{0,2proof,\theta}\right)$$ Eq. 7.1

The values for the retention factors at 811°C are obtained by linear interpolation. — Table 7.1

$k_{0,2proof,\theta}$ = 0,377

$k_{u,\theta}$ = 0,322

$k_{E,\theta}$ = 0,610

$g_{2\theta}$ = 0,353

Thus

$f_{2,\theta}$ = 0,377 × 220 + 0,353 × (0,322 × 530 – 0,377 × 220)

= 113,9 N/mm^2

$k_{2,\theta}$ = 113,9/220 = 0,518

Partial safety factor

$\gamma_{M,fi}$ = 1,0 — Section 7.1

Buckling resistance — Section 7.4.3

$N_{b,fi,t,Rd} = \chi_{z,fi}\, A_g\, k_{0,2proof,\theta}\, f_y / \gamma_{M,fi}$ — Eq. 7.8

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$\chi_{fi} = \dfrac{1}{\varphi_\theta + \sqrt{\varphi_\theta^{\,2} - \bar{\lambda}_\theta^{\,2}}}$ but $\leq 1{,}0$ — Eq. 7.10

$\varphi_\theta = 0{,}5\left(1 + \alpha\left(\bar{\lambda}_\theta - \bar{\lambda}_0\right) + \bar{\lambda}_\theta^{\,2}\right)$ — Eq. 7.11

$\bar{\lambda}_{z,\theta} = \bar{\lambda}_z\left[k_{0,2\mathrm{proof},\theta}/k_{E,\theta}\right]^{0,5} = 1{,}492 \times (0{,}377/0{,}610)^{0,5} = 1{,}173$ — Eq. 7.12

For flexural buckling of a hollow section, $\alpha = 0{,}49$ and $\bar{\lambda}_0 = 0{,}4$ — Table 5.1

$\varphi_{z,\theta} = 0{,}5\left(1 + 0.49(1.173 - 0.4) + 1.173^2\right) = 1{,}377$

$\chi_{z,fi} = \dfrac{1}{1{,}377 + \sqrt{1{,}377^2 - 1{,}173^2}} = 0{,}477$

$N_{b,fi,t,Rd} = 0{,}477 \times 1500 \times 0{,}377 \times 220/1{,}0 = 59{,}3$ kN

$N_{fi,Ed} = 13{,}0$ kN , buckling resistance of member is OK

Axial compression and bending moment

The following expression for a class 1 cross section must be satisfied

$$\frac{N_{fi,Ed}}{\chi_{min,fi}\left(A_g k_{0,2proof,\theta}\dfrac{f_y}{\gamma_{M,fi}}\right)} + \frac{k_y M_{y,fi,Ed}}{M_{y,fi,\theta,Rd}} + \frac{k_z M_{z,fi,Ed}}{M_{z,fi,\theta,Rd}} \leq 1$$ — Eq. 7.24

In which

$$k_y = 1 - \frac{\mu_y N_{fi,Ed}}{\chi_{y,fi} A_g k_{0,2,proof,\theta}\dfrac{f_y}{\gamma_{M,fi}}} \leq 3$$ — Eq. 7.28

$\mu_y = \left(1{,}2\beta_{M,y} - 3\right)\bar{\lambda}_{y,\theta} + 0{,}44\beta_{M,y} - 0{,}29 \leq 0{,}8$ — Eq. 7.29

$\bar{\lambda}_y = 0{,}866$ — Sheet 4

$\bar{\lambda}_{y,\theta} = \bar{\lambda}_y\left[k_{0,2proof,\theta}/k_{E,\theta}\right]^{0,5} = 0{,}866 \times (0{,}377/0{,}610)^{0,5} = 0{,}681$ — Eq. 7.12

Assume the column is fixed at the base, a triangular bending moment distribution occurs and $\beta_M = 1{,}8$ — Table 7.3

$\mu_y = (1{,}2 \times 1{,}8 - 3) \times 0{,}681 + 0{,}44 \times 1{,}8 - 0{,}29$

$= -0{,}070$

$\varphi_{y,\theta} = 0{,}5\left(1 + 0{,}49(0{,}681 - 0{,}4) + 0{,}681^2\right) = 0{,}801$

$\chi_{y,fi} = \dfrac{1}{0{,}801 + \sqrt{0{,}801^2 - 0{,}681^2}} = 0{,}818$

$$k_{\mathrm{y}} \quad = \quad 1-\frac{(-0{,}07)\times 13{,}0\times 10^{3}}{0{,}818\times 1500\times 0{,}377\times \dfrac{220}{1{,}00}} \qquad = \quad 1{,}009 < 3{,}0$$

Interaction expression:

$$\frac{N_{\mathrm{fi,Ed}}}{\chi_{\mathrm{min,fi}}\left(A_{g}\,k_{0{,}2\mathrm{proof},\theta}\,\dfrac{f_{\mathrm{y}}}{\gamma_{\mathrm{M,fi}}}\right)} + \frac{k_{\mathrm{y}}M_{\mathrm{y,fi,Ed}}}{M_{\mathrm{y,fi,\theta,Rd}}}$$

Eq. 7.13

$$M_{\mathrm{y,fi,\theta,Rd}} = k_{2,\theta}\left(\frac{\gamma_{\mathrm{M0}}}{\gamma_{\mathrm{M},fi}}\right)M_{\mathrm{Rd}} = 0{,}518\times\left(\frac{1{,}1}{1{,}0}\right)\times 8{,}75 = 4{,}99\ \ \mathrm{kNm}$$

$$\frac{13{,}0\times 10^{3}}{0{,}477\times 1500\times 0{,}377\times \dfrac{220}{1{,}0}} + \frac{1{,}009\times 1{,}82}{4.99} \ = \ 0{,}219 + 0{,}368 \ = \ 0{,}587$$

$0{,}587 < 1{,}00$

Thus section is OK in fire conditions for combined axial load and bending