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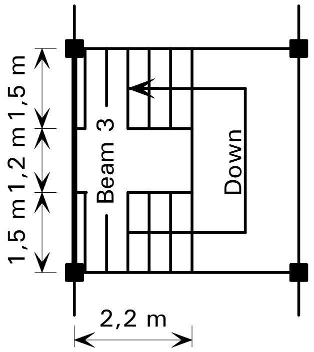
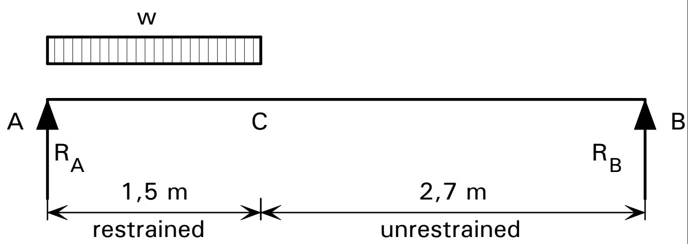
CALCULATION SHEET

DESIGN EXAMPLE 9 - BEAM WITH UNRESTRAINED COMPRESSION FLANGE

Design a staircase support beam. The beam is a single section channel, simply supported between columns. The flight of stairs lands between A and C and provides restraint to the top flange of this part of the beam. The top flange is unrestrained between B and C. The overall span of the beam is taken as 4,2 m.





Actions

Assuming the beam carries the load from the first run of stairs to the landing only:

Permanent actions (G): Load on stairs 1,0 kN/m^2 = (1,0 × 2,2) = 2,2 kN/m
Self weight of beam 0,13 kN/m

Variable actions (Q): Load on stairs 4 kN/m^2 = (4,0 × 2,2) = 8,8 kN/m

Load case to be considered (ultimate limit state):

$$\sum_{j\geq 1}\gamma_{G,j}G_{k,j} + \gamma_{Q,1}Q_{k,1} + \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i}$$ Eqn. 2.3

As there is only one variable action ($Q_{k,1}$) the last term in the above expression does not need to be considered in this example.

$\gamma_{G,j}$ = 1,35 (unfavourable effects) Section 2.3.2

$\gamma_{Q,1}$ = 1,5

Factored actions:

Permanent action:	Load on stairs	= 1,35 × 2,2	= 2,97 kN/m
	Self weight of beam	= 1,35 × 0,13	= 0,17 kN/m
Variable action	Load on stairs	= 1,5 × 8,8	= 13,2 kN/m

Structural analysis

Reactions at support points

$R_A + R_B$ = (2,97 + 13,2) × 1,5 + 0,17 × 4,2

= 24,97 kN

Taking moments about A

$$R_B = \frac{1{,}5 \times 16{,}17 \times 0{,}75 + 0{,}17 \times 4{,}2 \times (4{,}2/2)}{4{,}2} = 4{,}69 \text{ kN}$$

$$\Rightarrow R_A = 24{,}97 - 4{,}69 = 20{,}28 \text{ kN}$$

Maximum bending moment occurs at a distance $1{,}5\left(1 - \frac{1{,}5}{2 \times 4{,}2}\right) = 1{,}23$ m from A

$$M_{max} = 20{,}28 \times 1{,}23 - 16{,}17 \times \frac{1{,}23^2}{2} - 0{,}17 \times \frac{1{,}23^2}{2} = 12{,}58 \text{ kNm}$$

Maximum shear occurs at A

$F_{Sd} = 20{,}28$ kN

Material properties

Use material grade 1.4401

0,2% proof stress = 220 N/mm^2 — Table 3.1

Take f_y as the 0,2% proof stress = 220 N/mm^2 — Section 3.2.4

E = 200 000 N/mm^2 and G = 76 900 N/mm^2 — Section 3.2.4

Try a 200 × 75 channel section, thickness = 5 mm

Cross section properties

$I_y = 9{,}456 \times 10^6$ mm^4 $\quad W_{el,y} = 94{,}56 \times 10^3$ mm^3

$I_z = 0{,}850 \times 10^6$ mm^4 $\quad W_{pl,y} = 112{,}9 \times 10^3$ mm^3

$I_w = 5085 \times 10^6$ mm^4 $\quad A_g = 1650$ mm^2

$I_t = 1{,}372 \times 10^4$ mm^4

Classification of the cross-section

$\varepsilon = 1{,}01$ — Table 4.2

Assume conservatively that $c = h - 2t = 200 - 10 = 190$ mm for web

Web subject to bending: $\frac{c}{t} = \frac{190}{5} = 38$ — Table 4.2

For Class 1, $\frac{c}{t} \leq 56\varepsilon$, therefore web is Class 1

Outstand flange subject to compression: $\frac{c}{t} = \frac{75}{5} = 15$ — Table 4.2

For Class 3, $\frac{c}{t} \leq 11{,}9\varepsilon = 12{,}0$, therefore outstand flange is Class 4

Therefore, overall classification of cross-section is Class 4

Calculation of effective section properties

Calculate reduction factor ρ for cold formed outstand elements

$$\rho = \frac{1}{\bar{\lambda}_p} - \frac{0{,}231}{\bar{\lambda}_p^2} \quad \text{but} \leq 1$$ Eq. 4.1b

$$\bar{\lambda}_p = \frac{\bar{b}/t}{28{,}4\varepsilon\sqrt{k_\sigma}} \qquad \text{where } \bar{b} = c = 75\text{mm}$$ Eq. 4.2

Assuming uniform stress distribution within the compression flange, Table 4.4

$$\psi = \frac{\sigma_2}{\sigma_1} = 1$$

$$\Rightarrow k_\sigma = 0{,}43$$ Table 4.4

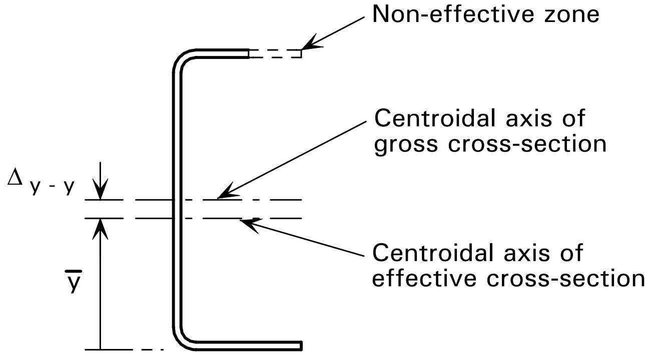
$$\bar{\lambda}_p = \frac{75/5}{28{,}4 \times 1{,}01 \times \sqrt{0{,}43}} = 0{,}797$$

$$\rho = \frac{1}{0{,}797} - \frac{0{,}231}{0{,}797^2} = 0{,}891$$

$$c_{eff} = 0{,}891 \times 75 = 66{,}8 \text{ mm}$$ Table 4.4

$$A_{eff} = A_g - (1-\rho)ct = 1650 - (1 - 0{,}891) \times 75 \times 5 = 1609 \text{ mm}^2$$

Calculate shift of neutral axis of section under bending



$$\bar{y} = \frac{A_g \times \frac{h}{2} - (1-\rho) \times c \times t \times \left(h - \frac{t}{2}\right)}{A_{eff}} = \frac{1650 \times \frac{200}{2} - (1 - 0{,}891) \times 75 \times 5 \times \left(200 - \frac{5}{2}\right)}{1609}$$

$$\bar{y} = 97{,}53 \text{ mm}$$

Shift of neutral axis position, $\Delta_{y-y} = \dfrac{h}{2} - \bar{y} = \dfrac{200}{2} - 97{,}53 = 2{,}47$ mm

Calculate $I_{\text{eff,y}}$

$$I_{\text{eff,y}} = \left(I_y - \frac{(1-\rho)ct^3}{12} - (1-\rho)ct\left(\frac{h}{2} - \frac{t}{2}\right)^2 - A_{eff}\,\Delta^2_{y-y} \right)$$

$$I_{\text{eff,y}} = 9{,}456 \times 10^6 - \frac{(1-0{,}891)\times 75 \times 5^3}{12} - (1-0{,}891)\times 75 \times 5 \times (100-2{,}5)^2 - 1609 \times 2{,}47^2$$

$$= 9{,}06 \times 10^6 \text{ mm}^4$$

$$W_{\text{eff,y}} = \frac{I_{\text{eff,y}}}{\dfrac{h}{2} + \Delta_{\text{y-y}}} = \frac{9{,}06 \times 10^6}{\dfrac{200}{2} + 2{,}47} = 88{,}4 \times 10^3 \text{ mm}^3$$

Shear lag

Section 4.4.2

Shear lag may be neglected provided that $b_0 \le L_e/50$ for outstand elements

L_e = distance between points of zero moment = 4200 mm

$L_e/50$ = 84 mm, b_0 = 75 mm , therefore shear lag can be neglected

Flange curling

Section 4.4.3

prEN 1993-1-3:2004 Clause 5.4(2) Eq. 5.3a

$$u = \frac{2\sigma_a^2 b_s^4}{E^2 t^2 z}$$

σ_a = average longitudinal stress in flange = 220 N/mm^2 (maximum possible value)

b_s = (75 – 5) = 70 mm

z = (100 – 2,5) = 97,5 mm

$$\therefore u = \frac{2 \times 220^2 \times 70^4}{200000^2 \times 5^2 \times 97{,}5} = 0{,}024 \text{ mm}$$

Flange curling can be neglected if $u < 0{,}05 \times 200 = 10$ mm

prEN 1993-1-3:2004 Clause 5.4(1)

Therefore flange curling is negligible

Partial safety factors

The following partial safety factors are used throughout the design example:

γ_{M0} = 1,1 and γ_{M1} = 1,1

Table 2.1

Moment resistance of cross-section — Section 4.7.4

For a class 4 cross section

$M_{c,Rd} = W_{eff,min} f_y / \gamma_{M0}$ — Eq. 4.29

$M_{c,Rd} = \frac{88{,}4 \times 10^3 \times 220}{1{,}1 \times 10^6} = 17{,}7$ kNm

Design moment = 12,58 kNm, ∴ cross-section moment resistance is OK

Cross-section resistance to shear — Section 4.7.5

$V_{pl,Rd} = A_v \left(f_y / \sqrt{3} \right) / \gamma_{M0}$ — Eq. 4.30

A_v = h × t = 200 × 5 = 1000 mm^2

$V_{Rd} = \frac{1000 \times 220}{\sqrt{3} \times 1{,}1 \times 1000} = 115{,}5$ kN

Design shear force = 20,28 kN, therefore shear resistance of cross-section is OK

Check that shear resistance is not limited by shear buckling

Assume that $h_w = h - 2t = 200 - 10 = 190$ mm

$\frac{h_w}{t} = \frac{190}{5} = 38$, shear buckling resistance needs to be checked if $\frac{h_w}{t} \geq \frac{52\varepsilon}{\eta} = 43{,}2\varepsilon$ — Section 5.4.3

∴ Shear resistance is not limited by shear buckling.

Resistance to lateral torsional buckling — Section 5.4.2

Compression flange of beam is laterally unrestrained between B and C. Check this portion of beam for lateral torsional buckling.

$M_{b,Rd} = \chi_{LT} W_{eff,y} f_y / \gamma_{M1}$ for a Class 4 cross-section — Eq. 5.8

$W_{eff,y} = 88.4 \times 10^3$ mm^3

$\chi_{LT} = \frac{1}{\varphi_{LT} + \left[\varphi_{LT}^2 - \bar{\lambda}_{LT}^2 \right]^{0,5}} \leq 1$ — Eq. 5.9

$\varphi_{LT} = 0{,}5 \left(1 + \alpha_{LT} \left(\bar{\lambda}_{LT} - 0{,}4 \right) + \bar{\lambda}_{LT}^2 \right)$ — Eq. 5.10

$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$ — Eq. 5.11

Determine the elastic critical moment (M_{cr}) — Appendix B

Section B.1

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left(\left[\left(\frac{k_z}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(k_z L)^2 G I_t}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j \right)^2 \right]^{1/2} - \left(C_2 z_g - C_3 z_j \right) \right)$$

C is simply supported, while *B* approaches full fixity. Assume most conservative case: $k_z = k_w = 1{,}0$.

C_1, C_2 and C_3 are determined from consideration of bending moment diagram and end conditions.

From bending moment diagram, $\psi = 0$

$\Rightarrow C_1 = 1{,}77,\ C_2 = 0$ and $C_3 = 1{,}00$ — Table B.1

$z_j = 0$ for a cross-section with equal flanges

$$M_{cr} = 1{,}77 \times \frac{\pi^2 \times 200000 \times 0{,}850 \times 10^6}{(1{,}00 \times 2700)^2} \times$$

$$\left(\left[\left(\frac{1{,}00}{1{,}00}\right)^2 \frac{5085 \times 10^6}{0{,}850 \times 10^6} + \frac{(1{,}00 \times 2700)^2 \times 76900 \times 1{,}372 \times 10^4}{\pi^2 \times 200000 \times 0{,}850 \times 10^6}\right]^{0,5}\right)$$

$M_{cr} = 41{,}9$ kNm

$$\bar{\lambda}_{LT} = \sqrt{\frac{88{,}4 \times 10^3 \times 220}{41{,}9 \times 10^6}} = 0{,}68$$ — Eq. 5.11

Using imperfection factor $\alpha_{LT} = 0{,}34$ for cold formed sections — Section 5.4.2

$$\varphi = 0{,}5\left(1 + 0{,}34(0{,}68 - 0{,}4) + 0{,}68^2\right) = 0{,}779$$

$$\chi_{LT} = \frac{1}{0{,}779 + \left[0{,}779^2 - 0{,}68^2\right]^{0,5}} = 0{,}863$$

$M_{b,Rd} = 0{,}863 \times 88.4 \times 10^3 \times 220 \times 10^{-6} / 1{,}1$

$= 15{,}3$ kNm

From bending moment diagram, maximum moment in unrestrained portion of beam = 12,0 kNm

Thus member has adequate resistance to lateral torsional buckling.

Deflection

Section 5.4.6

Load case (serviceability limit state): $\sum_{j\geq1} G_{k,j} + Q_{k,1} + \sum_{i\geq1} \psi_{0,i} Q_{k,i}$ — Eq. 2.8

As there is only one variable action ($Q_{k,1}$) the last term in the above expression does not need to be considered in this example.

Secant modulus is used for deflection calculations - thus it is necessary to find the maximum stress due to unfactored permanent and variable actions.

The secant modulus $E_S = \left(\dfrac{E_{S1} + E_{S2}}{2} \right)$, — Appendix C

Where

$$E_{s,i} = \frac{E}{1 + 0{,}002 \dfrac{E}{\sigma_{i,Ed,ser}} \left(\dfrac{\sigma_{i,Ed,ser}}{f_y} \right)^n} \quad \text{and } i = 1{,}2$$

From structural analysis calculations the following were found:

Maximum moment due to permanent actions = 1,90 kNm

Maximum moment due to imposed actions = 6,68 kNm

Total moment due to unfactored actions = 8,58 kNm

Section is Class 4, therefore W_{eff} is used in the calculations for maximum stress in the member.

Assume, conservatively that the stress in the tension and compression flange are approximately equal, i.e. $E_{S1} = E_{S2}$.

The following constants are used to determine the secant moduli:

For grade 1.4401 stainless steel, n (longitudinal direction) = 7,0 — Table C.1

Serviceability design stress, $\sigma_{i,Ed,ser} = \dfrac{M_{max}}{W_{eff,y}} = \dfrac{8{,}58 \times 10^6}{88{,}4 \times 10^3} = 97{,}1 \text{ N/mm}^2$

$$E_{s,i} = \frac{200000}{1 + 0{,}002 \times \dfrac{200000}{97{,}1} \times \left(\dfrac{97{,}1}{220} \right)^7} = 197\ 348 \text{ N/mm}^2$$

Maximum deflection due to patch loading occurs at a distance of approximately 1,9 m from support A.

Deflection at a distance x from support A due to patch load extending a distance a from support A is given by the following formulae:

When $x \geq a$ $\quad \delta = \dfrac{waL^4}{24aE_S I} n^2 \left[2m^3 - 6m^2 + m(4 + n^2) - n^2 \right]$ — Steel Designer's Manual (5th Ed)

Where $m = x/L$ and $n = a/L$

When x = 1,9 m, and a = 1,5 m: m = 1,9/4,2 = 0,452, n = 1,5/4,2 = 0,357

Patch load (permanent+variable unfactored actions) w = 11,0 kN/m

Uniform load (permanent action) w = 0,128 kN/m

Deflection due to patch loads at a distance of 1,9 m from support A, δ_1

$$\delta_1 = \frac{11000 \times 1{,}5 \times 4200^4}{24 \times 1500 \times 197348 \times 9{,}06 \times 10^6} \times 0{,}357^2 \left[2 \times 0{,}452^3 - 6 \times 0{,}452^2 + 0{,}452\left(4 + 0{,}357^2\right) - 0{,}357^2 \right]$$

= 7,09 mm

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CALCULATION SHEET

Job No.	OSM 466	Sheet	8 of 8	Rev	B
Job Title	ECSC Stainless Steel Valorisation Project				
Subject	Design Example 9 – Beam with unrestrained compression flange				
Client	ECSC	Made by	SMH	Date	Aug 2001
		Checked by	NRB	Date	Dec 2001
		Revised by	MEB	Date	April 2006

Deflection at midspan due to self weight of beam, δ_2

$$\delta_2 = \frac{5}{384} \times \frac{(w \times L)L^3}{E_S I} = \frac{5}{384} \times \frac{(0,128 \times 10^3 \times 4,2) \times 4200^3}{197348 \times 9,06 \times 10^6} = 0,29 \text{ mm}$$

Total deflection $\approx \delta_2 + \delta_2 = 7,09 + 0,29 = 7,38$ mm

$$\delta_{\text{limiting}} \quad \frac{\text{span}}{250} = \frac{4200}{250} = 16,8 \text{ mm}$$

Therefore deflection is acceptably small.