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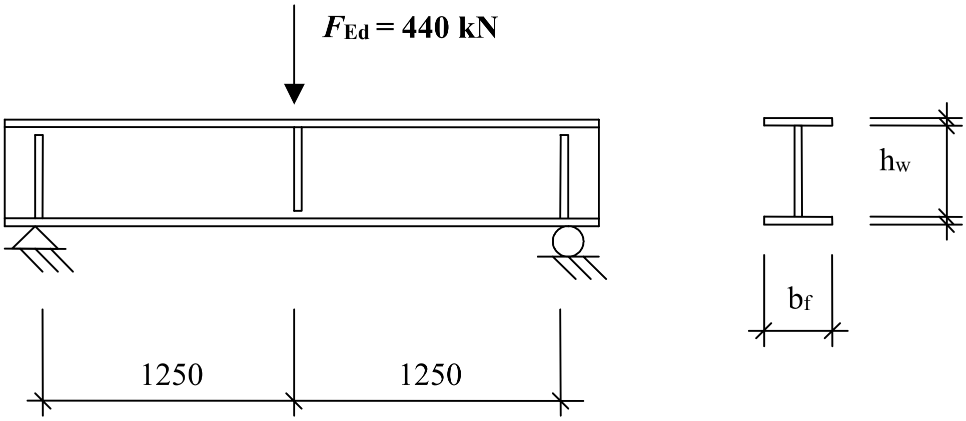
CALCULATION SHEET

DESIGN EXAMPLE 7 – SHEAR RESISTANCE OF PLATE GIRDER

Design a plate girder with respect to shear resistance. The girder is a simply supported I-section with a span according to the figure below. The top flange is laterally restrained.



Use material grade 1.4462, hot rolled. — Table 3.1

f_y = 460 N/mm^2 — Section 3.2.4

E = 200 000 N/mm^2

Try a cross section with

Flanges:	12 × 200 mm^2
Web:	4 × 500 mm^2
Stiffeners:	12 × 98 mm^2
Weld throat thickness:	4 mm

Structural analysis

Maximum shear and bending moment are obtained as

$$V_{Ed} = \frac{F_{Ed}}{2} = \frac{440}{2} = 220\,\text{kN}$$

$$M_{Ed} = \frac{F_{Ed}L}{4} = \frac{440 \times 2{,}5}{4} = 275\,\text{kNm}$$

Partial safety factors

γ_{M0} = 1,1 — Table 2.1

γ_{M1} = 1,1

Classification of the cross-section

Section 4.3

$$\varepsilon = \sqrt{\frac{235}{460}\frac{200}{210}} = 0{,}698$$

Table 4.2

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Web, subject to bending — Table 4.2

$$\frac{c}{t\varepsilon} = \frac{500-2\times\sqrt{2}\times 4}{4\times 0{,}698} = 175 > 74{,}8$$, therefore the web is Class 4.

Flange, subject to compression — Table 4.2

$$\frac{c}{t\varepsilon} = \frac{200-4-2\times\sqrt{2}\times 4}{2\times 12\times 0{,}698} = 11{,}0 \le 11{,}0$$, therefore the compression flange is Class 3.

Thus, overall classification of cross-section is Class 4.

Shear resistance — Section 5.4.3

The shear buckling resistance requires checking when $h_w / t_w \ge \frac{23}{\eta}\varepsilon\sqrt{k_\tau}$ for vertically stiffened webs.

$a/h_w = 1250/500 > 1$, hence

$$k_\tau = 5{,}34 + 4\left(\frac{h_w}{a}\right)^2 = 5{,}34 + 4\left(\frac{500}{1250}\right)^2 = 5{,}98$$ Eq. 5.16a

EN 1993-1-4 recommended value for $\eta = 1{,}2$ — Section 5.4.3

$$h_w/t_w = \frac{500}{4} = 125 \ge \frac{23}{1{,}2}0{,}698\sqrt{5{,}98} = 32{,}7$$

Therefore the shear buckling resistance has to be checked. It is obtained as

$$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd} \le \frac{\eta f_{yw} h_w t_w}{\sqrt{3}\,\gamma_{M1}} = \frac{1{,}2\times 460\times 500\times 4}{\sqrt{3}\times 1{,}1} = 579{,}47 \text{ kN}$$ Eq. 5.12a

$$V_{bw,Rd} = \frac{\chi_w f_{yw} h_w t_w}{\sqrt{3}\,\gamma_{M1}}$$ Eq. 5.12b

$$\chi_w = \eta = 1{,}2 \quad \text{for } \bar{\lambda}_w \le 0{,}60/\eta = 0{,}5$$ Eq. 5.13a

$$\chi_w = 0{,}11 + \frac{0{,}64}{\bar{\lambda}_w} - \frac{0{,}05}{\bar{\lambda}_w^2} \quad \text{for } \bar{\lambda}_w > 0{,}60/\eta = 0{,}5$$ Eq. 5.13b

$$\bar{\lambda}_w = \left(\frac{h_w}{37{,}4 t_w\,\varepsilon\sqrt{k_\tau}}\right)$$ Eq. 5.15

$$\bar{\lambda}_w = \left(\frac{500}{37{,}4\times 4\times 0{,}698\times\sqrt{5{,}98}}\right) = 1{,}958 > 0{,}60/\eta = 0{,}5$$

Hence the contribution from the web is obtained as:

$$\chi_w = 0{,}11 + \frac{0{,}64}{1{,}958} - \frac{0{,}05}{1{,}958^2} = 0{,}424$$

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$$V_{\mathrm{bw,Rd}} = \frac{\chi_{\mathrm{w}} f_{\mathrm{yw}} h_{\mathrm{w}} t_{\mathrm{w}}}{\sqrt{3}\gamma_{\mathrm{M1}}} = \frac{0{,}424 \times 460 \times 500 \times 4}{\sqrt{3} \times 1{,}1} = 204{,}74 \text{ kN}$$

The contribution from the flanges may be utilised if the flanges are not fully utilised in withstanding the bending moment. The bending resistance of a cross section consisting of the flanges only is obtained as — Section 5.4.3

$$M_{\mathrm{f,Rd}} = 12 \times 200 \times \frac{460}{1{,}1} \times (500 + 12) = 513{,}86\,\text{kNm}$$

$M_{\mathrm{f,Rd}} > M_{\mathrm{Ed}} = 275$ kNm, therefore the flanges can contribute to the shear buckling resistance.

$$V_{\mathrm{bf,Rd}} = \frac{b_{\mathrm{f}} t_{\mathrm{f}}^2 f_{\mathrm{yf}}}{c\gamma_{M1}} \left[1 - \left[\frac{M_{\mathrm{Ed}}}{M_{\mathrm{f,Rd}}}\right]^2\right]$$ Eq. 5.17

$$c = a\left[0{,}17 + \frac{3{,}5\, b_{\mathrm{f}}\, t_{\mathrm{f}}^2\, f_{\mathrm{yf}}}{t_{\mathrm{w}}\, h_{\mathrm{w}}^2\, f_{\mathrm{yw}}}\right] \quad \text{but} \quad \frac{c}{a} \le 0{,}65$$

$$= 1250 \times \left[0{,}17 + \frac{3{,}5 \times 200 \times 12^2 \times 460}{4 \times 500^2 \times 460}\right] = 338\,\text{mm} < 0.65 \times 1250 = 812\,\text{mm}$$

$$V_{\mathrm{bf,Rd}} = \frac{200 \times 12^2 \times 460}{338 \times 1{,}1} \times \left[1 - \left[\frac{275}{513{,}86}\right]^2\right] = 25{,}43\,\text{kN}$$

$$V_{\mathrm{b,Rd}} = V_{\mathrm{bw,Rd}} + V_{\mathrm{bf,Rd}} = 230{,}17\text{ kN} \le 579{,}47\,\text{kN}$$

Transverse stiffeners — Section 5.4.5

The transverse stiffeners have to be checked for crushing and flexural buckling using $\alpha = 0{,}49$, $\bar{\lambda}_0 = 0{,}2$. An effective cross section consisting of the stiffeners and parts of the web is then used. The part of the web included is $11\varepsilon t_w$ wide, therefore the cross section of the transverse stiffener is Class 3.

$a / h_{\mathrm{w}} = 1250 / 500 = 2{,}5 \ge \sqrt{2}$, hence the second moment of area of the intermediate stiffener has to fulfil — Eq. 5.37

$$I_{\mathrm{st}} \ge 0{,}75 h_{\mathrm{w}} t_{\mathrm{w}}^3 = 0{,}75 \times 500 \times 4^3 = 24000\,\text{mm}^4$$ Eq. 5.37

$$I_{\mathrm{st}} = 2 \times \frac{(11 \times 0{,}698 \times 4) \times 4^3}{12} + \frac{12 \times 200^3}{12} = 8{,}00 \times 10^6\,\text{mm}^4$$, hence fulfilled.

The crushing resistance is obtained as

$$N_{\mathrm{c,Rd}} = A_{\mathrm{s}} f_{\mathrm{y}} / \gamma_{\mathrm{M0}}$$ Eq. 4.25

$$A_{\mathrm{s}} = (12 \times 200 + 11 \times 0{,}698 \times 4 \times 2) = 2461{,}42\,\text{mm}^2$$

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Job No.		Sheet	4 of 6	Rev	B
Job Title	ECSC Stainless Steel Valorisation Project				
Subject	Design Example 7 – Shear resistance of plate girder				
Client	ECSC	Made by	AO	Date	June 2002
		Checked by	AT	Date	Oct 2002
		Revised by	MEB	Date	April 2006

$N_{c,Rd} = 2461{,}42 \times 460 / 1{,}1 = 1029{,}32\ \text{kN}$

The flexural buckling resistance is obtained as

$N_{b,Rd} = \chi A_s f_y / \gamma_{M1}$ — Eq. 5.2a

$$\chi = \frac{1}{\varphi + \left[\varphi^2 - \bar{\lambda}^2\right]^{0,5}} \le 1$$ Eq. 5.3

$$\varphi = 0{,}5\left(1 + \alpha\left(\bar{\lambda} - \bar{\lambda}_0\right) + \bar{\lambda}^2\right)$$ Eq. 5.4

$$\bar{\lambda} = \frac{L_{cr}}{i}\frac{1}{\pi}\sqrt{\frac{f_{yw}}{E}}$$ Eq. 5.5a

$L_{cr} = 0{,}75 h_w = 0{,}75 \times 500 = 375\ \text{mm}$ — Section 5.4.5

$$\bar{\lambda} = \frac{375}{\sqrt{\frac{8 \times 10^6}{2461{,}42}}}\frac{1}{\pi}\sqrt{\frac{460 \times 1}{200000}} = 0{,}100$$

$$\varphi = 0{,}5 \times \left(1 + 0{,}49 \times (0{,}100 - 0{,}2) + 0{,}100^2\right) = 0{,}481$$

$$\chi = \frac{1}{0{,}481 + \left[0{,}481^2 - 0{,}100^2\right]^{0,5}} = 1{,}05 > 1 \quad \Rightarrow \chi = 1{,}0$$

Since $N_{b,Rd} = N_{c,Rd} > N_{Ed}$, the transverse stiffeners are sufficient.

Interaction shear and bending

If the utilization of shear resistance, expressed as the factor $\bar{\eta}_3$, exceeds 0,5, the combined effect of bending and shear has to be checked. — Section 5.4.3

$$\bar{\eta}_3 = \frac{V_{Ed}}{V_{bw,Rd}} \le 1{,}0$$ Eq. 5.23

$$\bar{\eta}_3 = \frac{220}{204{,}74} = 1{,}075 > 0{,}5$$, therefore interaction has to be considered.

The condition is

$$\bar{\eta}_1 + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right)\left(2\bar{\eta}_3 - 1\right)^2 \le 1{,}0 \text{ for } \bar{\eta}_1 \ge \frac{M_{f,Rd}}{M_{pl,Rd}}$$ Eq. 5.21

Where:

$$\bar{\eta}_1 = \frac{M_{Ed}}{M_{pl,Rd}}$$ Eq. 5.22

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Calculation of effective cross-section properties.

The flanges are Class 3 and hence fully effective.

The depth of the web has to be reduced with the reduction factor ρ, welded web.

$$\rho = \frac{0{,}772}{\bar{\lambda}_p} - \frac{0{,}125}{\bar{\lambda}_p^{\,2}} \leq 1$$ Eq. 4.1a

$$\bar{\lambda}_p = \frac{\bar{b}/t}{28{,}4\varepsilon\sqrt{k_\sigma}} \quad \text{where } \bar{b} = d = 500 - 2\times 4\times\sqrt{2} = 488{,}68 \text{ mm}$$ Eq. 4.2

Assuming linearly varying, symmetric stress distribution within the web,

$$\psi = \frac{\sigma_2}{\sigma_1} = -1$$

$$\Rightarrow k_\sigma = 23{,}9$$ Table 4.3

$$\bar{\lambda}_p = \frac{488{,}68/4}{28{,}4\times 0{,}698\times\sqrt{23{,}9}} = 1{,}26$$

$$\rho = \frac{0{,}772}{1{,}26} - \frac{0{,}125}{1{,}26^2} = 0{,}534 \leq 1$$

$$b_{eff} = \rho\, b_c = \rho\, \bar{b}/(1-\psi) = 0{,}534\times 488{,}68/(1-(-1)) = 130{,}48$$ Table 4.3

$$b_{e1} = 0{,}4 b_{eff} = 0{,}4\times 130{,}48 = 52{,}19 \text{ mm}$$ Table 4.3

$$b_{e2} = 0{,}6 b_{eff} = 0{,}6\times 130{,}48 = 78{,}29 \text{ mm}$$

Calculate effective section modulus under bending.

e_i is taken as positive from the centroid of the upper flange and downwards.

$$A_{eff} = \sum_i A_i = b_f t_f \times 2 + b_{e1}t_w + b_{e2}t_w + (h_w/2)t_w = 6321{,}92 \text{ mm}^2$$

$$e_{eff} = \frac{1}{A_{eff}}\sum_i A_i e_i = \frac{1}{A_{eff}}\left[\, b_f t_f(0) + b_f t_f(h_w + t_f)\right] + \left[b_{e1}t_w(0{,}5(b_{e1} + t_f))\right.$$
$$\left. + \; b_{e2}t_w(0.5(h_w + t_f) - b_{e2}/2) + (h_w/2)t_w(0{,}75h_w + 0{,}5t_f)\;\right] = 266{,}44 \text{ mm}$$

$$I_{eff} = \sum_i I_i + \sum_i A_i(e_{eff} - e_i)^2 = 2\times\frac{b_f t_f^3}{12} + \frac{t_w b_{e1}^3}{12} + \frac{t_w b_{e2}^3}{12} + \frac{t_w (h_w/2)^3}{12}$$
$$+ \; b_f t_f (e_{eff} - 0)^2 + b_f t_f\left[e_{eff} - (h_w + t_f)\right]^2 + b_{e1}t_w\left[e_{eff} - 0{,}5(b_{e1} + t_f)\right]^2$$
$$+ \; b_{e2}t_w\left[e_{eff} - 0{,}5(h_w + t_f + b_{e2})\right]^2 + (h_w/2)t_w\left[e_{eff} - (0{,}75h_w + 0{,}5t_f)^2\right]^4$$
$$= 3{,}459\times 10^8 \text{ mm}^4$$

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$$\bar{\eta}_1 = \frac{M_{\mathrm{Ed}}}{M_{\mathrm{pl,Rd}}}$$

$M_{\mathrm{f,Rd}}$ = 513,86 kNm (Sheet 3)

$M_{\mathrm{pl,Rd}}$ is the plastic resistance of the cross-section.

$$M_{\mathrm{pl,Rd}} = M_{f,Rd} + \frac{t_w h_w{}^2 f_y}{4\gamma_{\mathrm{M0}}} = 513{,}86 + \frac{4\times 500^2\times 460}{4\times 1{,}1\times 10^6} = 618{,}40\,\mathrm{kNm}$$

<u>Evaluate conditions</u>

M_{Ed} = 275 kNm, hence:

$$\bar{\eta}_1 = \frac{275}{618{,}40} = 0{,}44 \le 1{,}0 \text{ OK}$$ Eq. 5.22

$\bar{\eta}_1$ fulfils its condition. Now it remains to check the interaction.

$$\bar{\eta}_1 + \left(1-\frac{M_{\mathrm{f,Rd}}}{M_{\mathrm{pl,Rd}}}\right)\left(2\bar{\eta}_3-1\right)^2 = 0{,}44+\left(1-\frac{513{,}86}{618{,}40}\right)\left(\left(2\times 1{,}075\right)-1\right)^2 = 0{,}664 < 1{,}0$$

It therefore follows that under the conditions given, the resistance of the plate girder is sufficient with respect to shear, bending as well as interaction between shear and bending.