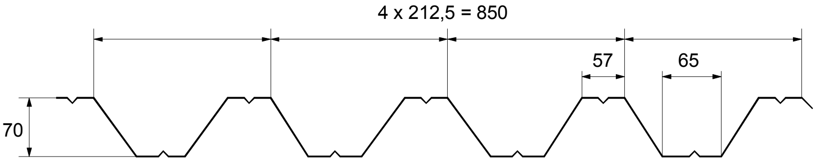
DESIGN EXAMPLE 3 – DESIGN OF A TWO-SPAN TRAPEZOIDAL ROOF SHEETING

This example considers the design of a two-span trapezoidal roof sheeting. The material is grade 1.4401 stainless steel and the material thickness is 0,6 mm. The dimensions of the cross section are shown below.



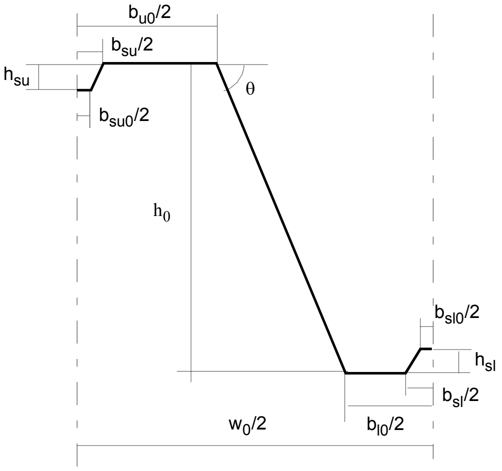
The example shows the following design tasks:
- determination of effective section properties at the ultimate limit state
- determination of the bending resistance of the section
- determination of the resistance at the intermediate support
- determination of deflections at serviceability limit state.

This example refers to prEN 1993-1-3:2005 and adopts its symbols and terminology. Reference should be made to prEN 1993-1-3 for a full description of the design procedures and associated figures.

Design data

Spans	L	=	2900 mm	
Width of supports	s_s	=	100 mm	
Design load	Q	=	1,4 kN/m^2	
Design thickness	t	=	0,6 mm	
Yield strength	f_{yb}	=	240 N/mm^2	Table 3.1
Modulus of elasticity	E	=	200 000 N/mm^2	Section 3.2.4
Partial safety factor	γ_{M0}	=	1,1	Table 2.1
Partial safety factor	γ_{M1}	=	1,1	Table 2.1

Symbols and detailed dimensions used in calculations are shown in the figure below. The position of the cross section is given so that in bending at the support the upper flange is compressed.



Centre line dimensions

$h_0 = 70$ mm
$w_0 = 212{,}5$ mm
$b_{u0} = 65$ mm
$b_{l0} = 57$ mm
$b_{su} = 20$ mm
$b_{su0} = 8$ mm
$h_{su} = 6$ mm
$b_{sl} = 20$ mm
$b_{sl0} = 8$ mm
$h_{sl} = 6$ mm

Internal radius of the corners $r = 3$ mm

Angle of the web:

$$\theta = \operatorname{atan}\left|\frac{h_0}{0{,}5(w_0 - b_{u0} - b_{l0})}\right| = 57{,}1 \text{ deg}$$

EFFECTIVE SECTION PROPERTIES AT THE ULTIMATE LIMIT STATE — Section 4.4

Check on maximum width-to-thickness ratios:

$h_0 / t = 117$ is less than $400 \sin\theta = 336$ and — Table 4.1

$b_{l0} / t = 95$ is less than 400. — Table 4.1

Location of the centroidal axis when the web is fully effective

Effective width of the compressed flange:

$$b_p = \frac{b_{u0} - b_{su}}{2} = 22{,}5 \text{ mm} \qquad \varepsilon = \left[\frac{235}{f_y}\frac{E}{210\,000}\right]^{0,5} = 0{,}966$$ — Section 4.4.1

$$k_\sigma = 4 \qquad \bar{\lambda}_p = \frac{b_p / t}{28{,}4\varepsilon\sqrt{k_\sigma}} = 0{,}684$$

$$\rho = \frac{0{,}772}{\bar{\lambda}_p} - \frac{0{,}125}{\bar{\lambda}_p^{\,2}} = 0{,}862$$

Because $\rho < 1$, $b_{eff,u} = \rho b_p = 19{,}4$ mm — Eq. 4.1a

Reduced thickness of the flange stiffener: — Section 4.5.3

$$t_{su} = \frac{\sqrt{h_{su}^{\,2} + \left(\frac{b_{su} - b_{su0}}{2}\right)^2}}{h_{su}} t = 0{,}849 \text{ mm}$$

$$A_s = (b_{eff,u} + b_{su0})t + 2h_{su}t_{su} = 26{,}6\ \text{mm}^2$$ Fig. 4.3

$$e_s = \frac{b_{su0}h_{su}t + 2h_{su}\dfrac{h_{su}}{2}t_{su}}{A_s} = 2{,}23\ \text{mm}$$

$$I_s = 2\left(15t^2{e_s}^2\right) + b_{su0}t(h_{su} - e_s)^2 + 2h_{su}t_{su}\left(\frac{h_{su}}{2} - e_s\right)^2 + 2\left(\frac{15t^4}{12}\right) + \frac{b_{su0}t^3}{12} + 2\frac{t_{su}{h_{su}}^3}{12}$$ Fig. 4.3

$$= 159{,}53\ \text{mm}^4$$

$$b_s = 2\sqrt{{h_{su}}^2 + \left(\frac{b_{su} - b_{su0}}{2}\right)^2} + b_{su0} = 25{,}0\ \text{mm}$$

$$l_b = 3{,}07\left(I_s{b_p}^2\frac{2b_p + 3b_s}{t^3}\right)^{1/4} = 251\ \text{mm}$$ Eq. 4.9

$$s_w = \sqrt{\left(\frac{w_0 - b_{u0} - b_{l0}}{2}\right)^2 + {h_0}^2} = 83{,}4\ \text{mm}$$

$$b_d = 2b_p + b_s \qquad k_{w0} = \sqrt{\frac{s_w + 2b_d}{s_w + 0{,}5b_d}} = 1{,}37$$ Eq. 4.10 and 4.11

$l_b / s_w = 3{,}01$ Because $l_b / s_w > 2$, $k_w = k_{w0} = 1{,}37$

$$\sigma_{cr,s} = \frac{4{,}2k_wE}{A_s}\sqrt{\frac{I_st^3}{4{b_p}^2(2b_p + 3b_s)}} = 515\ \text{N/mm}^2$$ Eq. 4.3

$$\bar{\lambda}_d = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}} = 0{,}683$$

Because $0{,}65 < \bar{\lambda}_d < 1{,}38$, $\chi = 1{,}47 - 0{,}723\bar{\lambda}_d = 0{,}98$ Eq. 4.15

$$t_{red,u} = \chi t = 0{,}588\ \text{mm}$$

The distance of neutral axis from the compressed flange:

$$t_l = \frac{\sqrt{{h_{sl}}^2 + \left(\dfrac{b_{sl} - b_{sl0}}{2}\right)^2}}{h_{sl}}t = 0{,}849\ \text{mm}$$

$$t_w = t / \sin\theta = 0{,}714\ \text{mm}$$

$e_i =$	$A_i =$
0	$0{,}5b_{\mathrm{eff,u}}\,t$
0	$0{,}5b_{\mathrm{eff,u}}\chi\,t$
$0{,}5h_{\mathrm{su}}$	$h_{\mathrm{su}}\chi t_{\mathrm{su}}$
h_{su}	$0{,}5b_{\mathrm{su0}}\chi\,t$
$0{,}5h_0$	$h_0 t_{\mathrm{w}}$
h_0	$0{,}5(b_{l0}-b_{sl})t$
$h_0-0{,}5h_{sl}$	$h_{sl}t_{sl}$
h_0-h_{sl}	$0{,}5b_{sl0}t$

$$A_{\mathrm{tot}} = \sum A_i = 87{,}5\ \mathrm{mm}^2$$

$$e_{\mathrm{c}} = \frac{\sum A_i e_i}{A_{\mathrm{tot}}} = 34{,}9\ \mathrm{mm}$$

Effective cross-section of the compression zone of the web — Section 4.4.1

$$\psi = -\frac{h_0-e_{\mathrm{c}}}{e_{\mathrm{c}}} = -1{,}006 \qquad k_\sigma = 5{,}98(1-\psi)^2 = 24{,}1$$ Table 4.3

$$b_{\mathrm{p}} = h_0/\sin\theta = 83{,}4\ \mathrm{mm} \qquad \bar{\lambda}_{\mathrm{p}} = \frac{b_{\mathrm{p}}/t}{28.4\varepsilon\sqrt{k_\sigma}} = 1{,}032$$

$$\rho = \frac{0{,}772}{\bar{\lambda}_{\mathrm{p}}} - \frac{0{,}125}{\bar{\lambda}_{\mathrm{p}}^2} = 0{,}630$$ Eq. 4.1a

Because $\rho < 1$, $b_{\mathrm{eff}} = \rho\dfrac{b_{\mathrm{p}}}{1-\psi} = 26{,}2\ \mathrm{mm}$ — Table 4.3

$s_{\mathrm{eff,1}} = 0{,}4b_{\mathrm{eff}} = 10{,}5\ \mathrm{mm}$ $\quad s_{\mathrm{eff,n}} = 0{,}6b_{\mathrm{eff}} = 15{,}7\ \mathrm{mm}$ — Table 4.3

Effective cross section properties per half corrugation

$$h_{\mathrm{eff,1}} = s_{\mathrm{eff,1}}\sin\theta \qquad h_{\mathrm{eff},n} = s_{\mathrm{eff},n}\sin\theta$$

$e_{\mathrm{eff},i} =$	$A_{\mathrm{eff},i} =$	$I_{\mathrm{eff},i}$
0	$0{,}5b_{\mathrm{eff,u}}t$	0
0	$0{,}5b_{\mathrm{eff,u}}\chi\,t$	0
$0{,}5h_{\mathrm{su}}$	$h_{\mathrm{su}}\chi\,t_{\mathrm{su}}$	$\chi t_{\mathrm{su}}h_{\mathrm{su}}^3/12$
h_{su}	$0{,}5b_{\mathrm{su0}}\chi\,t$	0
$0{,}5h_{\mathrm{eff,1}}$	$h_{\mathrm{eff,1}}t_{\mathrm{w}}$	$t_{\mathrm{w}}h_{\mathrm{eff,1}}^3/12$
$h_0-0{,}5(h_0-e_{\mathrm{c}}+h_{\mathrm{eff},n})$	$(h_0-e_{\mathrm{c}}+h_{\mathrm{eff},n})t_w$	$t_{\mathrm{w}}(h_0-e_{\mathrm{c}}+h_{\mathrm{eff},n})^3/12$
h_0	$0{,}5(b_{l0}-b_{sl})t$	0

$h_0 - 0{,}5h_{sl}$	$h_{sl}t_{sl}$	$t_{sl}h_{sl}^{\ 3}/12$
$h_0 - h_{sl}$	$0{,}5b_{sl0}t$	0

$A_{tot} = \sum A_{eff,i} = 78{,}2\ \text{mm}^2$ $\qquad e_c = \dfrac{\sum A_{eff,i}\, e_{eff,i}}{A_{tot}} = 37{,}20\ \text{mm}$

$I_{tot} = \sum I_{eff,i} + \sum A_{eff,i}\left(e_c - e_{eff,i}\right)^2 = 58\,400\ \text{mm}^2$

Optionally the effective section properties may also be redefined iteratively based on the location of the effective centroidal axis. *prEN 1993-1-3, clause 5.5.3.3(3)*

Bending strength per unit width (1 m) *Section 4.7.4*

$I = \dfrac{1000\ \text{mm}}{0{,}5\text{w}_0} I_{tot} = 549\,000\ \text{mm}^4$

$W_u = \dfrac{I}{e_c} = 14\,800\ \text{mm}^3$ $\qquad W_l = \dfrac{I}{h_0 - e_c} = 16\,800\ \text{mm}^3$

Because $W_u < W_l$, $\qquad W_{eff,min} = W_u = 14\,800\ \text{mm}^3$

$M_{c,Rd} = W_{eff,min} f_y / \gamma_{M0} = 3{,}22\ \text{kNm}$ *Eq. 4.29*

DETERMINATION OF THE RESISTANCE AT THE INTERMEDIATE SUPPORT

Web crippling strength *Section 5.4.4*

Here $\varphi = \theta$

$l_a = s_s$ and $\alpha = 0{,}15$ *prEN 1993-1-3, Eq. 6.19b and 6.20c*

$$R_{w,Rd} = \alpha t^2 \sqrt{f_{yb}E}\left(1 - 0{,}1\sqrt{\frac{r}{t}}\right)\left(0{,}5 + \sqrt{0{,}02\frac{l_a}{t}}\right)\left[2{,}4 + \left(\frac{\phi}{90\ \text{deg}}\right)^2\right]\frac{1}{\gamma_{M1}}\frac{1000\ \text{mm}}{0{,}5\text{w}_0}$$

prEN 1993-1-3, Eq. 6.18

$= 16{,}2\ \text{kN}$

Combined bending moment and support reaction

Factored actions per unit width (1m):

$\gamma_G = 1{,}35$ $\qquad \gamma_Q = 1{,}5$ $\qquad$ Self weight: $G = 70\ \text{N/m}^2$ *Section 2.3.2*

$q = \left(\gamma_G G + \gamma_Q Q\right) = 2{,}20\ \text{kN/m}$ *Eq. 2.3*

$M_{\mathrm{Ed}} = \dfrac{qL^2}{8} = 2{,}31\,\mathrm{kNm}$ $\qquad F_{\mathrm{Ed}} = \dfrac{5}{4}qL = 7{,}96\,\mathrm{kN}$

$\dfrac{M_{\mathrm{Ed}}}{M_{\mathrm{c,Rd}}} = 0{,}716$ $\qquad \dfrac{F_{\mathrm{Ed}}}{R_{\mathrm{w,Rd}}} = 0{,}491$ $\qquad \dfrac{M_{\mathrm{Ed}}}{M_{\mathrm{c,Rd}}} + \dfrac{F_{\mathrm{Ed}}}{R_{\mathrm{w,Rd}}} = 1{,}21$

Combined bending moment and support reaction satisfy the conditions:

$\dfrac{M_{\mathrm{Ed}}}{M_{\mathrm{c,Rd}}} \le 1$ $\qquad \dfrac{F_{\mathrm{ed}}}{R_{\mathrm{w,Rd}}} \le 1$ $\qquad \dfrac{M_{\mathrm{Ed}}}{M_{\mathrm{c,Rd}}} + \dfrac{F_{\mathrm{Ed}}}{R_{\mathrm{w,Rd}}} \le 1{,}25$ — prEN 1993-1-3, Eqs 6.28a-c

DETERMINATION OF DEFLECTIONS AT SERVICEABILITY LIMIT STATE (SLS)

Effective cross section properties

For serviceability verification the effective width of compression elements should be based on the compressive stress in element under the serviceability limit state loading. — prEN 1993-1-3, clause 5.5.1(4)

Maximum compressive stress in the effective section at SLS. A conservative approximation is made based on W_{u} determined above for ultimate limit state. — Section 2.3.4

$$M_{\mathrm{y,Ed,ser}} = \frac{(G+Q)L^2}{8} = 1{,}55\,\mathrm{kNm}$$

$$\sigma_{\mathrm{com,Ed,ser}} = \frac{M_{\mathrm{y,Ed,ser}}}{W_u} = 105\,\mathrm{N/mm^2}$$

The effective section properties are determined as before in ultimate limit state except that f_{yb} is replaced by $\sigma_{\mathrm{com,Ed,ser}}$ and the thickness of the flange stiffener is not reduced.

The results of the calculation are:

Effective width of the compressed flange	The flange is fully effective
Location of the centroidal axis when the web is fully effective	$e_{\mathrm{c}} = 34{,}1$ mm
Effective cross-section of the compression zone of the web	The web is fully effective
Effective part of the web	The web is reduced ($\rho = 0{,}88$)
Effective cross section properties per half corrugation	$A_{\mathrm{tot}} = 86{,}6\ \mathrm{mm^2}$
	$e_{\mathrm{c}} = 34{,}8$ mm
	$I_{\mathrm{tot}} = 63\ 700\ \mathrm{mm^4}$
Effective section properties per unit width (1 m)	$I = 600\ 000\ \mathrm{mm^4}$
	$W_{\mathrm{u}} = 17\ 300\ \mathrm{mm^4}$
	$W_l = 17\ 300\ \mathrm{mm^4}$

Determination of deflection

Appendix C

Secant modulus of elasticity corresponding to maximum value of the bending moment:

$$\sigma_{1,\mathrm{Ed,ser}} = \frac{M_{\mathrm{y,Ed,ser}}}{W_{\mathrm{u}}} = 89{,}5\ \mathrm{N/mm^2}$$

$$\sigma_{2,\mathrm{Ed,ser}} = \frac{M_{\mathrm{y,Ed,ser}}}{W_{l}} = 90{,}8\ \mathrm{N/mm^2}$$

$$E_{\mathrm{s},1} = \frac{E}{1+0{,}002\dfrac{E}{\sigma_{1,\mathrm{Ed,ser}}}\left(\dfrac{\sigma_{1,\mathrm{Ed,ser}}}{f_{\mathrm{yb}}}\right)^{n}} = 199\ \mathrm{kN/mm^2} \qquad n = 7{,}0$$

Appendix C
Table C.1

$$E_{\mathrm{s},2} = \frac{E}{1+0{,}002\dfrac{E}{\sigma_{1,\mathrm{Ed,ser}}}\left(\dfrac{\sigma_{2,\mathrm{Ed,ser}}}{f_{yb}}\right)^{n}} = 199\ \mathrm{N/mm^2}$$

$$E_{\mathrm{s,ser}} = \frac{E_{\mathrm{s},1}+E_{\mathrm{s},2}}{2} = 199\ \mathrm{N/mm^2}$$

Appendix C

Check of deflection:

As a conservative simplification, the variation of $E_{\mathrm{s.ser}}$ along the length of the member is neglected.

$$x = \frac{1+\sqrt{33}}{16}L = 1{,}22\ \mathrm{m} \quad \text{(location of max deflection)}$$

$$\delta = \frac{(G+Q)L^4}{48E_{\mathrm{s,ser}}I}\left(\frac{x}{L}-3\frac{x^3}{L^3}+2\frac{x^4}{L^4}\right) = 4{,}7\ \mathrm{mm}$$

The permissible deflection is $L/200 = 14{,}5\ \mathrm{mm}$, hence the calculated deflection is acceptable.

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